A tutorial for the Sliderule Activity

(http://activities.sugarlabs.org/en-US/sugar/addon/4222)

running on the Sugar operating system http://sugarlabs.org/ of the OLPC laptop,

targeted at year 8 to year 12 students

by Tony Forster,

History

The slide rule, also known colloquially as a slipstick, is a mechanical analogue computer. The slide rule is used primarily for multiplication and division, and also for functions such as roots, logarithms and trigonometry, but is not normally used for addition or subtraction. William Oughtred and others developed the slide rule in the 1600s based on the emerging work on logarithms by John Napier.

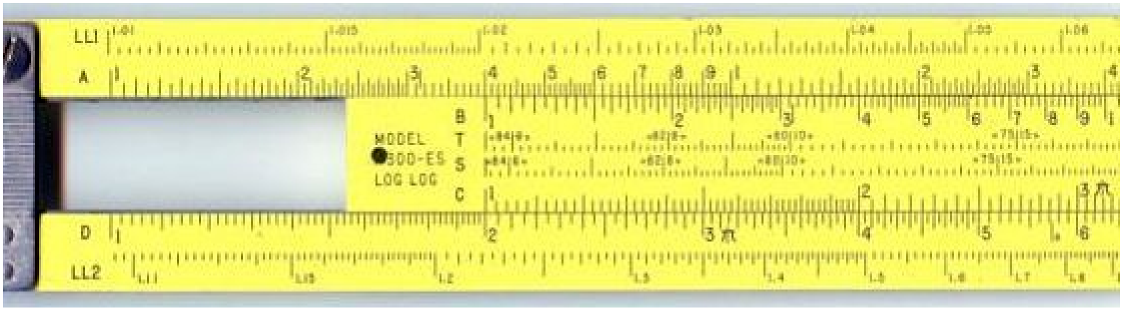

Image http://commons.wikimedia.org/wiki/File:Pocket_slide_rule.jpg

Before the advent of the pocket calculator, it was the most commonly used calculation tool in science and engineering. The use of slide rules continued to grow through the 1950s and 1960s even as digital computing devices were being gradually introduced; but around 1974 the electronic scientific calculator made it largely obsolete. (http://en.wikipedia.org/wiki/Sliderule)

The slide rule is still a useful way of thinking about mathematics and posing challenging problems.

Adding and subtracting

You can use your fingers for integer (whole number) addition, 2 fingers plus 3 fingers equals 1,2,3,4,5 fingers. For fractional numbers (rational numbers) you could use a centimetre ruler, measure out 2.3cm then another 1.2 cm and measure the total, 3.5cm.

Start the Sliderule Activity on your Sugar http://sugarlabs.org/ operating system.

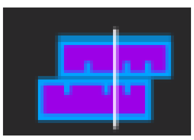

Fig 1 the Sliderule Activity icon

(If Sliderule is not installed on your Sugar operating system, you can get it at http://activities.sugarlabs.org/en-US/sugar/addon/4222)

Select the add/subtract slide rule

Fig 2

Slide the top scale till its 0 lines up with 2.3 on the bottom scale, then slide the slide rule's cursor, (the vertical line on the slide rule), to 1.2 on the top scale. Read the result at the cursor on the bottom scale.

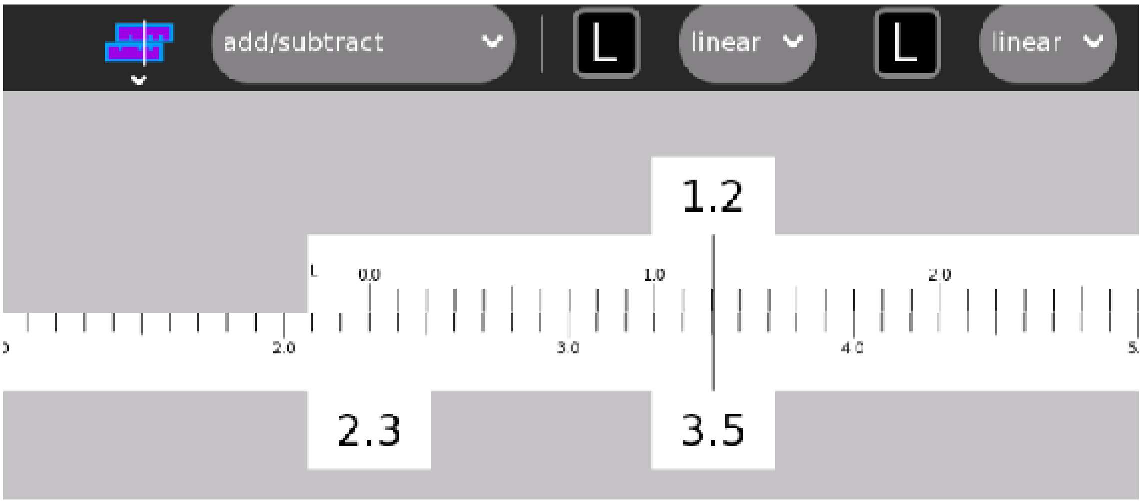

Fig 3 2.3 +1.2 =3.5

Did you know? The pointer on a computer screen is called a cursor. Cursors have been around for a lot longer than computers. A cursor is a moving placement or pointer that indicates a position. English-speakers have used the term with this meaning since the 16th century, for a wide variety of movable or mobile position-markers. The literal meaning of the original Latin word cursor expresses the idea of someone or something that runs. http://en.wikipedia.org/wiki/Cursor

This slide rule can be used for subtracting.

Q1 See if you can use the slide rule for calculating 4 - 1.5 (Answers at the end of this document)

Hint: Mark out 4 on the bottom scale and then go back 2.5 on the top scale.

Multiplication and division

The numbers 100 and 1000 can be represented as powers of 10. 100 is 10 x 10 or 10^2, 1000 is 10 x 10 x 10 = 10^3. 1 is 10^0 and numbers less than 1 have a negative index, 1/10 is 10^{-1} and 1/100 is 10^{-2}.

Fractional indices give numbers that are intermediate between 10,100,1000... For example, the square root of 10 is $10^{0.5}$ or $10^{(1/2)}$ or approximately 3.1623.

The multiplication of 100 by 1000 can be represented as $10^2 \times 10^3 = 10^{(2+3)} = 10^5$, to multiply, just add the indices. Another example: $1000 \times 0.1 = 10^3 \times 10^{-1} = 10^{(3-1)} = 10^2$.

To multiply, add the logarithms, to divide, subtract.

$1000/100 = 10^3/10^2 = 10^{(3-2)} = 10^1 = 10$

Q2 Calculate 100 x 1000/10 by arithmetic operations on the indices.

These indices are called the logarithms of the numbers. $Log_{10}(10)=1$ $Log_{10}(100)=2$ and $Log_{10}(1000)=3$

Note: The 10 in Log_{10} means that we are working with powers of 10, it is possible to construct logarithms on other bases, the most notable are natural logarithms, Log_e or Ln, which are based on powers of e (2.718) http://en.wikipedia.org/wiki/E_(mathematical_constant)

Ln(e)=1 Ln(e x e) =2 etc.

Q3 What is Log_{10} (0.01)

Multiplication

In Sliderule, change to the multiply/divide slide rule

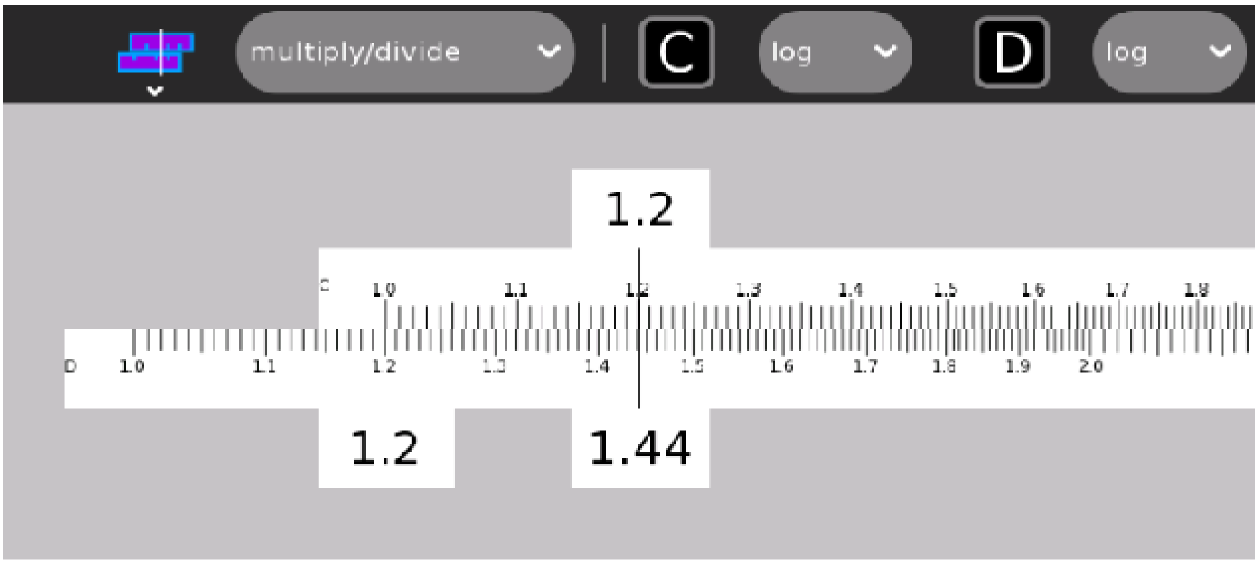

Fig 4   1.2 x 1.2 = 1.44

See how the C and D scales are compressed to their right end . Though the scales are marked with numbers, their distance along the scales are proportional to the logarithms of those numbers. When two distances are added, the logarithms of the numbers are added, if logarithms are added, numbers are multiplied.

In this case, the distance marked 1.2 on the upper scale is added to the distance marked 1.2 on the lower scale or

Log_{10} (1.2) + Log_{10} (1.2)

the resulting distance is Log_{10} (1.2 x 1.2) = Log_{10} (1.44) the result of the multiplication

1.2 x 1.2 = 1.44

is read off on the bottom scale.

Q4 Use the slide rule to calculate 1.5 x 2

Division

To divide you subtract the index, Fig 4 also represents 1.44 / 1.2 = 1.2

The log of 1.2 is subtracted from the log of 1.44

Q5 use the slide rule to calculate 3/2 =1.5

Hint: move 3 to the right on the D scale and come back left 2 on the C scale.

Overflow and underflow

Using the CI scale

If you try to calculate 3 x 4 = 12, the answer lies off the right end of the slide rule. Similarly calculating 3/4=0.75 lies off the left end of the slide rule. Here you can use the CI or inverse scale. To multiply you divide by the inverse and to divide you multiply by the inverse.

Change to the Divide/Multiply slide rule which has the CI scale as the top scale. Move the cursor to 3.0 on the bottom scale. Align the 4.0 on the top, inverse scale with the cursor. Read the result at the left end of the top scale.

The top scale is marked with the Log(1/number). We have subtracted Log(1/4) from Log(3). Subtracting Log(number) is the same of dividing by (number). We have calculated

3/(1/4) = 3 x 4 =12

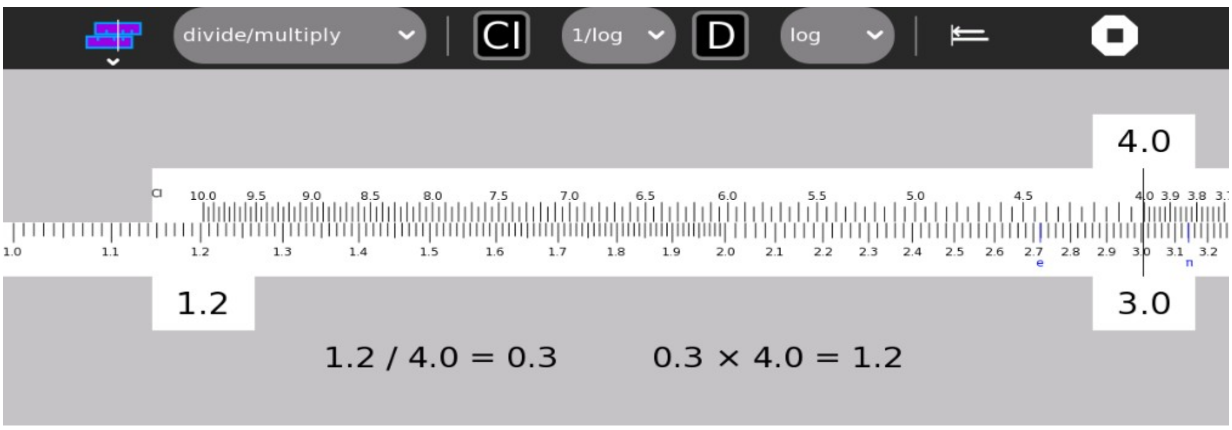

Fig 5, The CI scale, 3 x 4 = 12

To calculate 3/7.5 move to 3 on the lower scale, move further right 7.5 on the top scale

That is 3 x (1/7.5)=4

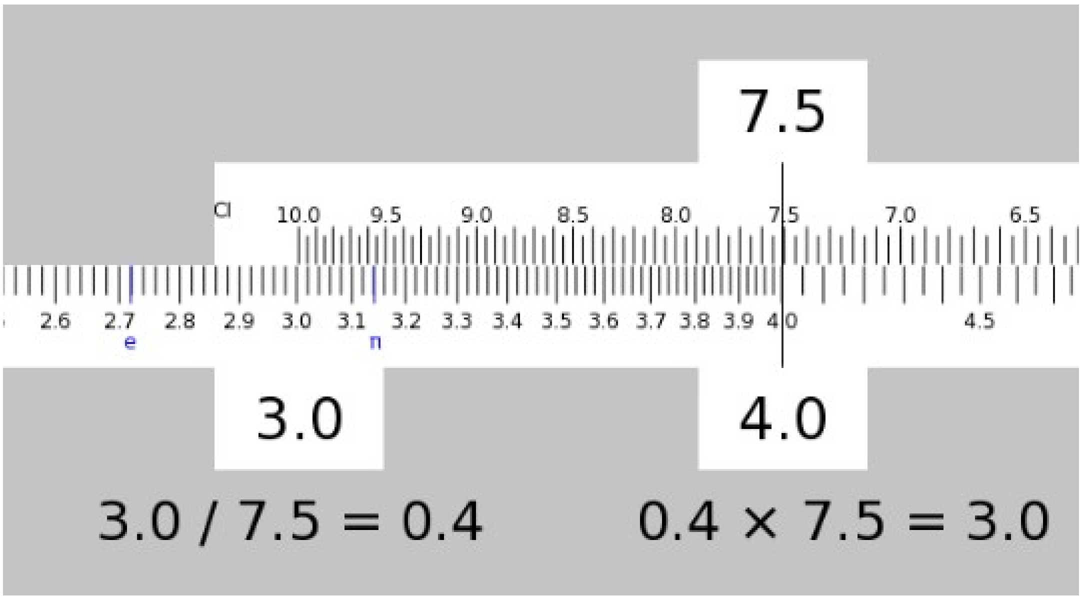

Figure 6, CI scale 3/7.5 = 4

Using the C/D scale

Consider, the slide rule, the distance from 1 on the left up to a number represents the log of that number. Below shown with a red arrow is the distance representing the number 5. The distance is proportional to log(5)

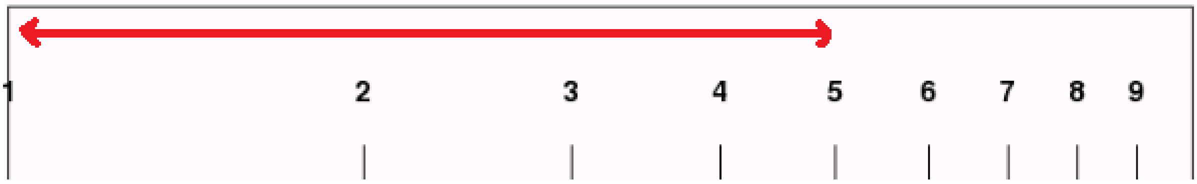

Figure 7, C or D scale Log 5

Consider the distance to the right of the number. What does that represent?

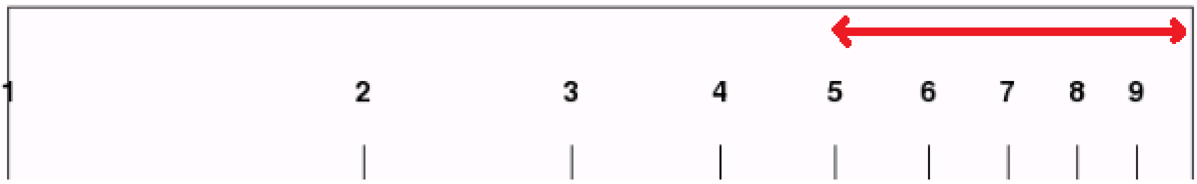

Figure 8, C or D scale Log (1/5)

The distance to the right of the number represents the inverse of 5, 1/5 or 0.2. This is because the full length of the rule represents the number 1 (not 10, **we ignore powers of 10** as is explained later). If we add the distances to multiply and the two distances add up to 1 then they must be reciprocals

5 x 1/5 = 1 or

log(5) + log(1/5) = log(1)

If the distance to the right of a number represents its reciprocal and you move left by that distance, you have divided by the reciprocal or multiplied by the number.

Consider the multiplication of 3 x 4 which we previously multiplied by using the CI scale to avoid falling off the end of the rule. This multiplication can be done on the C and D scales.

Move to 3 on the bottom scale, then move left by the distance between 4 and the right hand end of the scale, the result is 12

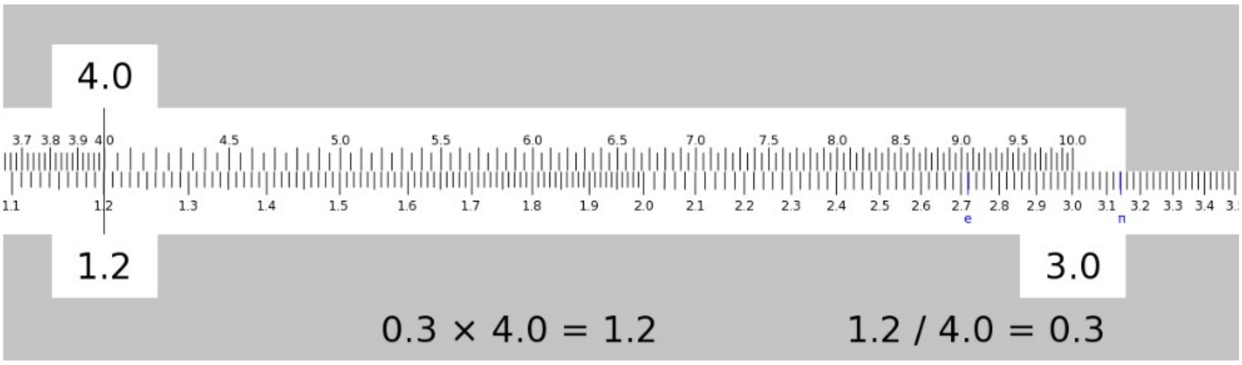

Figure 9, C and D scale 3 x 4 = 12

Powers of 10

We saw how the slide rule can be used to calculate with numbers 1<n<10, what if we wanted to calculate 12 x 12. Simply ignore the position of the decimal place, calculate and estimate the position of the decimal place in the answer. In the case of 12 x 12, calculate 1.2 x 1.2 = 1.44, then estimate the answer, 12 x 12 is a bit more than 100 so the answer is 144 (not 1.44 , 1440 or 14.4).

Q6 Calculate 37 x 19

Q7 Calculate 38/622

Other scales

Square/square root

Change to the square/square root slide rule.

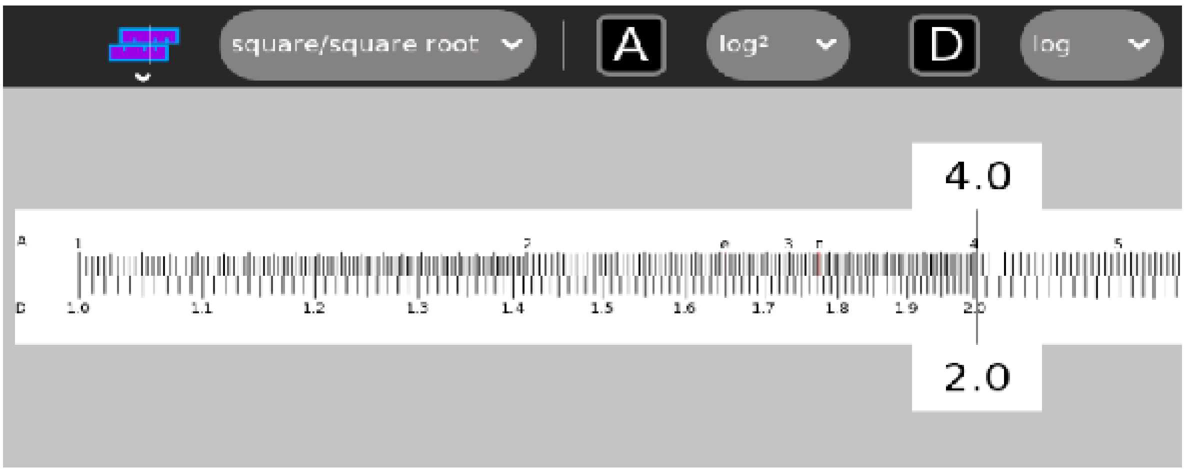

Fig 10 the square/square root slide rule

Scale A shows the square of the number on scale D

Q8 If the distance of scale D is proportional to the logarithm of the number, what is scale A?

Cube/cube root

Scale K can be used in a similar way for cubes.

Q9 What do you get if you read from scale A to scale K?

Sin/asin

Read the angle in degrees on the S scale and the sin on the D scale, for example sin(90)=1 and sin(45)=0.707 (remember you have to estimate the decimal point)

Q10 Note the slide rule only reads down to 5.7 degrees, how could you use the slide rule for smaller angles?

Hint: what do you know of sin(x) for small x

Q11 How can you calculate cos(x)

Hint: what is the relationship between sin and cos

Q12 If the D scale is proportional to the log, what is the S scale proportional to?

Tan/atan

Read the angle in degrees on the T scale and the sin on the D scale, for example tan(45)=1 and tan(30)=0.577 (remember you have to estimate the decimal point)

Q13 What do you get if you enter data on the S scale and look up the answer on the T scale?

Custom scales

Make a custom slide rule with linear L and log D scales

L is $\log_{10}(D)$

or D is 10^L

10^0 is 1 or $\mathrm{Log}_{10}(1)=0$

10^1 is 10 or $\mathrm{Log}_{10}(10)=1$

$10^{0.5}$ is 3.16 or root(10)

Q14 what do you get with L and A?

Q15 what is S and A?

Answers

Q1

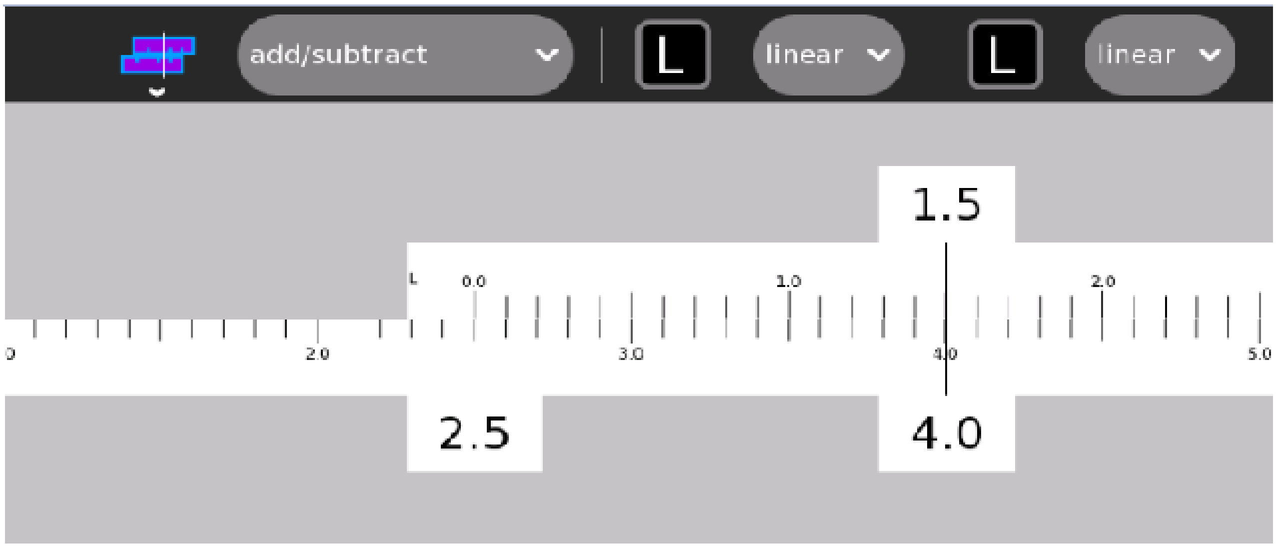

Fig 11, 4.0 - 1.5 = 2.5

Q2

100 x 1000/10

$=10^2 \times 10^3 / 10^1$

$=10^{(2+3-1)}$

$=10^4$

=10000

Q3

What is Log_{10} (0.01)

$0.01 = 10^{-2}$ so the index or logarithm is -2

Log_{10} (0.01)=-2

Q4

Use the slide rule to calculate 1.5 x 2

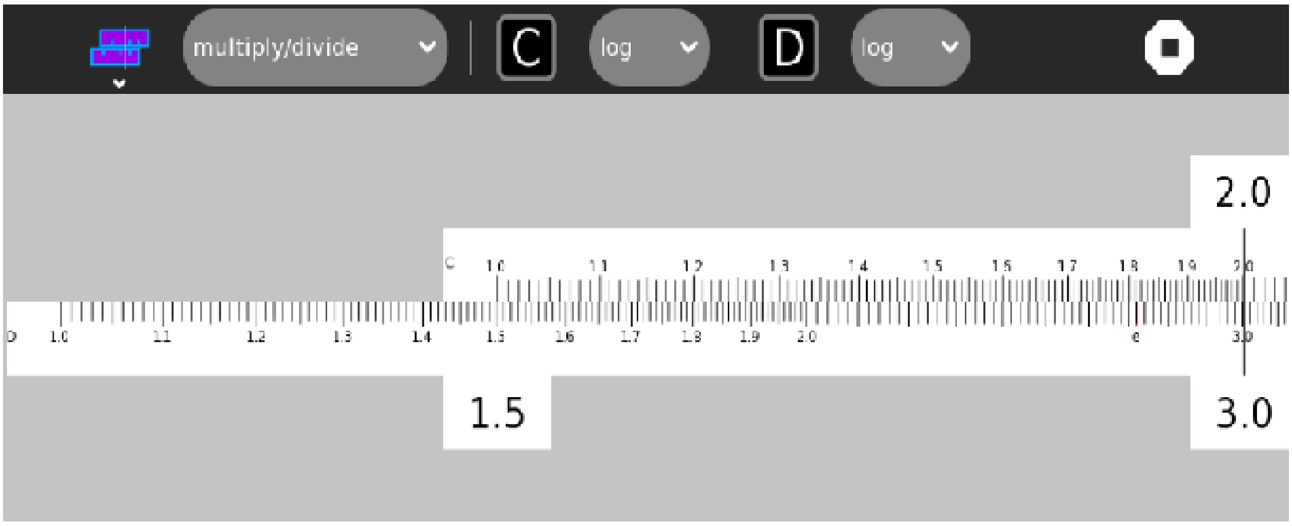

Fig 12,   1.5 x 2 = 3

Q5 use the slide rule to calculate 3/2 =1.5

Answer, it's the same as the diagram above, line up 3 on D and 2 on C, read the answer on the D scale

Q6 Calculate 37 x 19

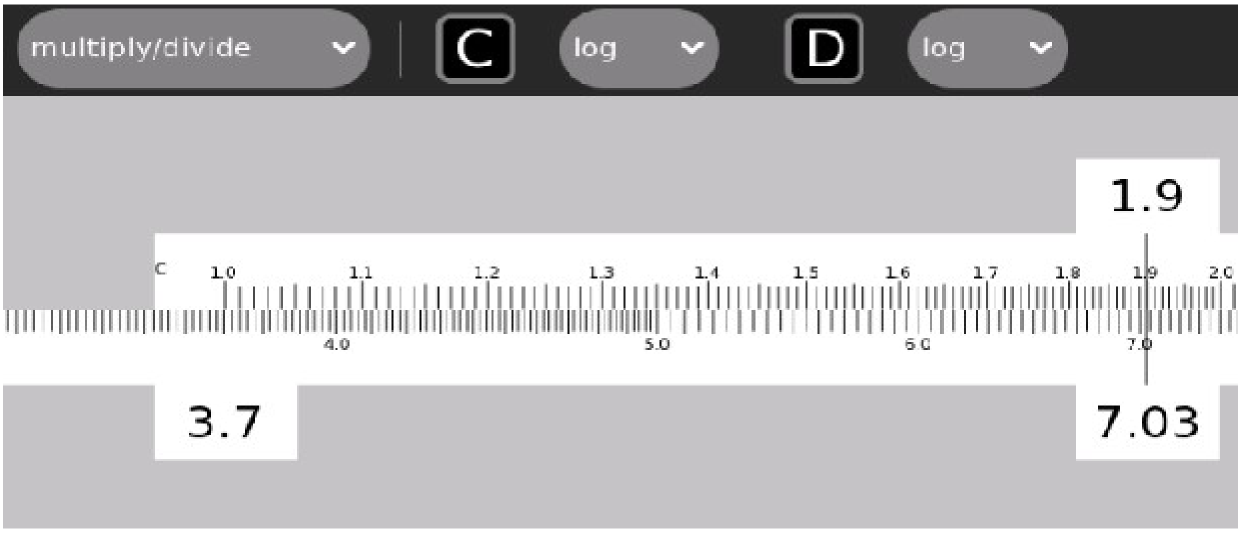

Fig 13,   3.7 x 1.9 = 7.03

estimating 40 x 20 = 800

so the answer must be 703

Q7 Calculate 38/622

Calculating 3.8/6.22 would underflow the LHS of the C,D scales so use the CI scale. Now its 3.8 multiplied by the reciprocal of 6.22. Move to 3.8 on the D scale and a further 6.22 right on the CI scale. Read the answer on the D scale, 0.611

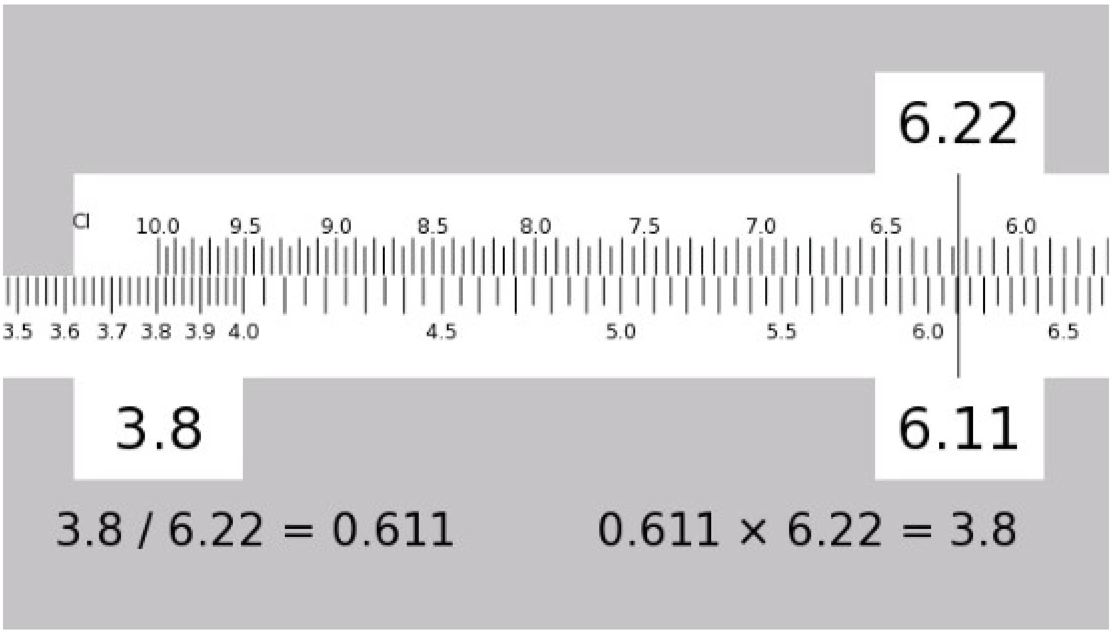

Fig 14,   38 / 622 = 7.03

estimating 40/600 = 2/30 = 0.066

so the answer is 0.0611

(the actual answer is 0.0610932475884244 but the slide rule is at best accurate to three decimal places)

Q8 If the distance of scale D is proportional to Log(n) then

distance of scale C is proportional to 0.5(Log(n))

Q9 What do you get if you read from scale A to scale K?

Scale A = scale D^2, scale K = scale D^3

Scale K = scale $A^{3/2}$ or $A^{1.5}$

Q10 Note the slide rule only reads down to 5.7 degrees, how could you use the slide rule for smaller angles?

Could use the fact that sin(x) ? x for small x in radians

5 degrees=5 x Pi/180 radians = 0.0873 radians

whereas sin(5degrees) = 0.0872

Q11 How can you calculate cos(x)

cos(x) = sin(90-x)

Q12 asin(log) ?

Q13 What do you get if you enter data on the S scale and look up the answer on the T scale?
Tan(asin(x))

Q14 what do you get with L and A?
$\log_{100}(x)$

Q15 S and A?
$\sin^2$